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Comparison of Numerical Integration with Trapesium and Romberg Methods Using Hypertext Preprocessor Program

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Abstract: The Trapesio and Romberg methods are two classic numerical methods for calculating integrals. The Trapesio method is a numerical integration method that uses an approach based on the area of a trapezoid, but is often less accurate for non-linear functions. In contrast, the Romberg method, which uses Richardson extrapolation, offers higher accuracy even in working on more complex mathematical functions. The purpose of this study is to compare the accuracy and speed of completion in numerical integration on various mathematical functions. The type of research used is pure experimental. Using the *Hypertext Preprocessor* (PHP) program, both methods are tested for accuracy and speed in solving the given problem. The results show that the Romberg method provides more accurate results than the Trapesium method with an average error of 0.979% in this study, where the result with the lowest error for the Romberg method is in the integral of the polynomial function with an error of 0%. In the case of the Trapesio method, the average error in this study is 9.497%, where the lowest error for the Trapesio method is in the exponential function integral with an error of 0.178%. However, the improvement of accuracy requires a fairly long program completion time on the Romberg method with an average completion time of 4,400 seconds, where the fastest completion time is 0,007 seconds. Meanwhile, in the Trapezium method, the time required in program execution is very fast with an average $3{,}13 \times 10^{-6}$ of seconds, where the fastest completion time is seconds.$1{,}19 \times 10^{-6}$

Keywords: Trapezium Method, Romberg Method, PHP, Error, Completion Time.

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Introduction

In the era of globalization, the development of science and technology, including mathematics, is very rapid. Mathematics, as a tool and foundation of various disciplines, helps us understand natural phenomena, develop business strategies, and design new technologies (Prasetya, 2016). With the advent of various applications and programming, complex mathematical calculations are now easier to solve, including through numerical methods. According to Firdaus, Amrullah, Wulandari, & Hikmah (2023), numerical calculations usually use the help of applications such as Matlab, Pascal, or Java because of their long calculations. One of the complex calculations in mathematics is an integral that often requires a numerical method. Suppose a partition P from an interval to n interval-sections using a point and suppose . At each section interval take a random point called the sample point for the i-part interval (Varberg, Purcell, & Rigdon, 2007). In some cases, there are integral calculations whose application is difficult to calculate using analytical methods (Erviana, Amrullah, Triutami & Subarinah, 2023). $[a,b] a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ $\Delta x_i = x_i - x_{i-1} [x_{i-1}, x_i], x_i f$ $[a,b]$.

According to Subarinah (2022), the numerical method for integration is an integral calculation of course based on approximate calculations. This calculation often results in errors called errors.

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Numerical integration becomes important when the solution by analytical means is difficult to implement due to the complexity of the calculus formulas used. Numerical integration is also widely applied in various fields such as science, economics, construction, and biology. For example, in population modeling, numerical methods help solve complex growth models. Complex mathematical models often take the form of non-linear functions. Non-linear models are difficult to solve with analytical methods to obtain their exact values (Sujaya, Prayitno, Kurnianti, & Sridana, 2024). There are many numerical integration methods that can be used to solve various difficult problems, including the Trapesium and Romberg methods. Both of these methods can be used to solve complex non-linear function integrals.

The Trapesium and Romberg methods are often used for their simplicity and flexibility, although each has different levels of accuracy and efficiency. The trapezoidal method replaces the curve curve of the function, the x-axis between and , is approached by the area of the trapezoidal plane below the line connecting the points and (Herfina, Amrullah, & Junaidi, 2019). According to $f(x)x = ax = b\ (a, f(a))(b, f(b))$Østerby (2005), the Norway mathematist Werner Romberg first described a systematic extrapolation procedure in the context of a numerical integration formula known as the trapezoidal rule, which is based on the values of the functions at the endpoints of each sub-interval and has an error similar to that of the equation. The Romberg method is a numerical integration method based on the Richardson extrapolation expansion resulting from the trapezoidal rule (Ernawati, Rahayu & Zuhairo, 2017).

Previous research by Prasetya (2016) used Matlab to compare the Trapesium and Gauss-Legendre methods, while Firdaus et al. (2023) used Pascal for the Simpson method. However, neither of these programming languages is ideal for web application development. Therefore, this study uses PHP, which is famous for being free, cross-platform, and fast in web access (Siswanto, 2021).

This study aims to compare the effectiveness of Trapesium and Romberg methods in numerical integration using PHP. PHP was chosen because of its advantages in data processing and ease of access, so the results of this research are expected to provide a more practical and easy-to-implement solution in solving numerical integration.

Method

This study is experimental research that aims to find out the comparison of errors of the numerical integral of the Trapesium and Romberg methods using PHP programming, as well as to find out the comparison of the completion time required from each method. The experimental research used is a pure experiment (true experimental). The algorithm used for the numerical integration of the Trapesium method and the Romberg method in this study is as follows:

Trapezium Method Algorithm:

1. Specifies the function to be integrated.$f(x)$
2. Define the lower and upper limits of the integration.$(a)(b)$
3. Determining the number of trapezoids.(n)
4. Determine the exact value of the integral function.
5. Determine the value of the numerical integration approach with the Trapesium method based on the Trapesium method table.
6. Calculate the proximity value, error and program execution time on the Trapesio method.

The flow diagram for the trapezoidal method in the PHP program is as follows:

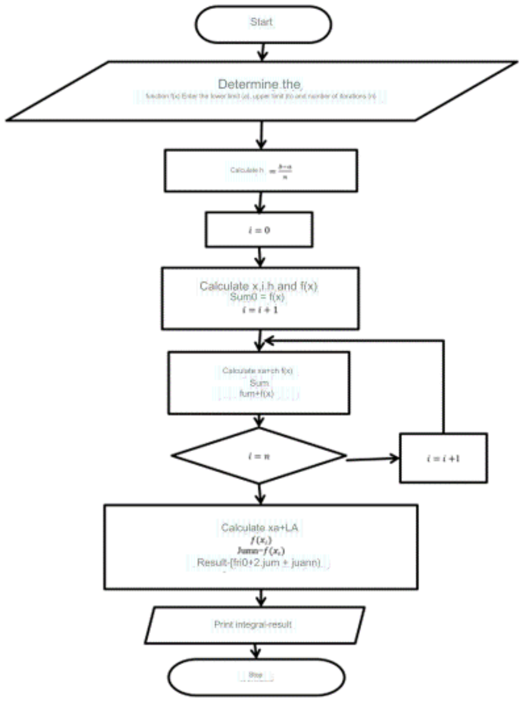

Figure 1. Flow diagram of the Trapesite method

Romberg Method Algorithm

1. Specifies the function to be integrated.$f(x)$
2. Define the lower and upper limits of the integration.$(a)(b)$
3. Specifies the iteration rate.(n)
4. Determine the exact value of the integral function.
5. Determining the value of the numeric integration approach with the Romberg method based on the Romberg iteration table.

6. Calculate the proximity value, error and execution time of the program on the Romberg method.

The flow diagram for the Romberg method in the PHP program is as follows:

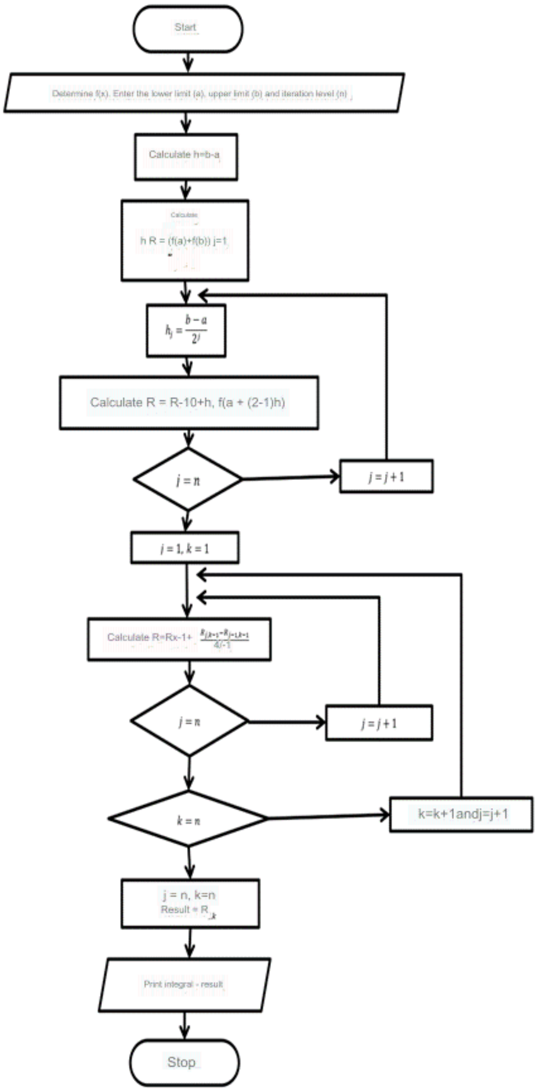

Figure 1. Flow diagram of the Romberg method

The steps taken in this study include:

1. Preparation, preparation by investigating the literature to collect data and information related to non-linear functions, the Trapesium method and the Romberg method, as well as the exact value of an integral definite function. By relying on a variety of sources, including books, notes, and other materials, to gain a comprehensive understanding
2. Application of the Trapesium and Romberg Method, the creation of programming in the form of PHP to complete numerical integration in mathematical functions using the Trapesium and Romberg methods.
3. Trial of the Trapesium and Romberg Method Program, the program trial was carried out on several functions that have obtained approximate values and exact values
4. Revision of the Trapesium and Romberg Method Program, revise the program if there are errors
5. Implement program code into non-linear functions to get approximation and execution time of programs
6. Analyzing the Results of the Program
7. Drawing conclusions.

Results and Discussion

The program code used in PHP is created following the algorithm of the Trapesium and Romberg methods. PHP programs are validated by adjusting the integral results obtained by the program with the results of simple functions that have previously been obtained approximate and exact values. Pay attention to Table 1.

Table 1. Program results

Function	Ex Ak	Trapecium Method			Romberg Method		
		Iteration	Approximation	Error (%)	Iteration	Approximation	Error (%)
$\int_0^2 \sin x \cos x \, dx$	0,4134	4	0,3784	8,4638	4	0,4134	0,00001
$\int_0^1 e^x dx$	1,7183	2	1,7539	2,07182	2	1,7182	0,00582
$\int_0^1 \frac{1}{x+1} \, dx$	0,6931	1	0,75	8,20949	1	0,6944	0,18756

Based on Table 1, the approximate values obtained in the Trapesium and Romberg methods are the same as the values that have been obtained previously. This causes the program to be said to be valid. The numerical integration of the Trapesio method and the Romberg method used for the collection of comparative data of approximate values and errors in non-linear functions is as follows:

1. $\int_{-2}^{1} x^5 + 3x^3 + 2x^2 + 9x + 7 \, dx$
2. $\int_0^4 x^{\frac{3}{2}} - 2x^2 + 5\sqrt{x} - 3 \, dx$
3. $\int_1^3 x^6 + 5x^5 - 4x^2 + 9x \, dx$
4. $\int_{-1}^{1} 2^x e^x \, dx$
5. $\int_1^4 \frac{e^{\sqrt{2x}}}{\sqrt{x}} \, dx$
6. $\int_{-1}^{0} x^2 e^{x^3} \, dx$
7. $\int_0^1 \cos 5x \sin 6x \, dx$
8. $\int_0^5 x \cos^2(x^2 - 9) \, dx$
9. $\int_0^2 \sin 9x \, \sin x \, dx$

The results of the numerical integration calculation of the Trapesium method and the Romberg method using PHP programs can be seen in Table 1.

Table 2. Research Results

Function	Exact	Iteration	Trapecium Method		Romberg Method	
			Approximation	Error (%)	Approximation	Error (%)
$\int_{-2}^{1} x^5 + 3x^3 + 2x^2 + 9x + 7\,dx$	$-8{,}25$	10	$-8{,}9223$	8,14909		
$\int_{0}^{4} x^{\frac{3}{2}} - 2x^2 + 5\sqrt{x} - 3\,dx$	$-15{,}2$	9	$-15{,}7048$	3,32105		
$\int_{-1}^{1} x^6 + 5x^5 - 4x^2 + 9x\,dx$	$-2{,}38096$	3	$-2{,}59076$	8,81157		
$\int_{-1}^{1} 2^x e^x\,dx$	3,1023	8	3,1485	1,48922		
$\int_{1}^{4} \frac{e^{\sqrt{2x}}}{\sqrt{x}}\,dx$	18,1098	5	18,1420	0,1778		
$\int_{-1}^{0} x^2 e^{x^3}\,dx$	0,2107	3	0,2072	1,66113		
$\int_{0}^{1} \cos 5x \sin 6x\,dx$	0,2751	7	0,2650	3,67193		
$\int_{0}^{5} x\cos^2(x^2 - 9)\,dx$	6,5621	5	7,7484	18,07805		
$\int_{0}^{2} \sin 9x \sin x\,dx$	$-0{,}0636$	9	$-0{,}03809$	40,11006		

In Table 2, the iteration rate used is always the same in the Trapesium and Romberg method programs. However, the approximate values obtained by the two methods are different, as are the errors in each function which are not always the same even though the iteration rate used is the same for different functions. (n) The results of the program using the Romberg method show an advantage in approaching the exact value of the integral of the non-linear function compared to the trapezoidal method. By the same iteration, the Romberg method provides approximation values that are more accurate and very close to the exact values of those integrals. The recursive approach used in the Romberg method allows for a significant increase in accuracy by increasing the rate of iteration used. So the Romberg method became a better choice for numerical integral calculations in cases where high accuracy was required. A comparison of the average errors obtained by the Trapesium and Romberg methods on polynomial, exponential and trigonometric functions can be seen in Table 3.

Table 3. Average Error

Function Type	**Average Error**	
	Trapezoidal method	**Romberg method**
Polynomial Function	6,761	0,114
Exponential Function	1,109	0,000157
Trigonometric Function	20,620	2,824
Average	9,497	0,979

Table 3 shows the mean errors in polynomial, exponential, trigonometric functions for each Trapesio and Romberg method based on the results obtained in Table 3. The comparison can be presented in the form of a bar chart as shown in Figure 3.

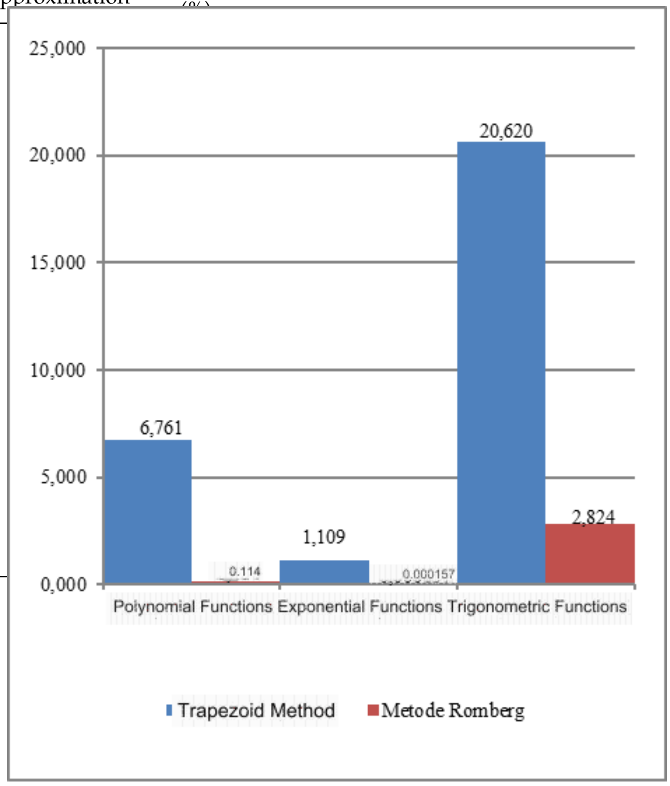

Figure 3. Comparison of average errors

Based on Figure 3, the results of the program using the Romberg method show an advantage in approaching the exact value of the integral of the non-linear function compared to the Trapesium method. By the same iteration, the Romberg method provides approximation values that are more accurate and very close to the exact values of those integrals. The recursive approach used in the Romberg method allows for a significant increase in accuracy by increasing the rate of iteration used. So the Romberg method became a better choice for numerical integral calculations in cases where high accuracy was required. Respectively, the mean error in the Trapesium and Romberg methods was 9.497% and 0.979%. In the Trapezium method, the lowest error is in the exponential function, i.e., 0.178%. Meanwhile, in the Romberg method, the lowest error is in the polynomial function, which is 0%.

In addition to comparing errors, researchers also compared the time it takes for computers to solve a given integral problem based on each method used. The functions used in comparing time are as follows:

1. $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_nx^0$
2. $\frac{ae^{\sqrt{bx}}}{\sqrt{cx}}$
3. $ax \cos bx \ \sin cx$

The results of the time calculation on the function with the value, the value change, the lower bound and the upper bound can be seen in Table 4.$a_0x^5 + a_1x^4 + a_2x^3 - a_3x^3 + a_4x + a_5n = 5a_i(a) = 0(b) = 10$

Table 4. Calculating time on the function $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_nx^0$

a_0	a_1	a_2	a_3	a_4	a_5	n	Iteration	Program execution time (second)	
								Trapecium Method	Romberg Method
1	2	0	-6	7	10	5	10	$1{,}91 \times 10^{-6}$	0.011
1	2	0	-6	7	10	5	13	$3{,}10 \times 10^{-6}$	0,065
1	2	4	-6	8	10	5	15	$4{,}05 \times 10^{-6}$	0,262
1	2	4	-6	8	10	5	17	$2{,}86 \times 10^{-6}$	1,035
1	3	0	-6	8	10	5	20	$4{,}05 \times 10^{-6}$	9.476
1	3	0	-6	8	10	5	21	$4{,}05 \times 10^{-6}$	16,453

Based on Table 4, the average execution time for the polynomial function is as follows:

1. In the second Trapezium method. $3{,}34 \times 10^{-6}$
2. On the Romberg method of seconds.4,544

The calculation of time on functions with lower and upper limits, where the values and at the function change can be seen in Table 5.$\frac{ae^{\sqrt{bx}}}{\sqrt{cx}}(a) = 1(b) = 5a, b\ c$

Table 5. Calculating time on the function $\frac{ae^{\sqrt{bx}}}{\sqrt{cx}}$

a	b	c	Iteration	Program execution time (second)	
				Trapecium Method	Romberg Method
3	5	7	10	$4{,}05 \times 10^{-6}$	0,011
4	5	7	13	$2{,}86 \times 10^{-6}$	0,071
5	6	7	15	$3{,}10 \times 10^{-6}$	0,394
5	8	7	17	$2{,}15 \times 10^{-6}$	1,030
6	7	8	20	$3{,}10 \times 10^{-6}$	8,710
6	9	8	21	$2{,}86 \times 10^{-6}$	16,994

Table 5 shows the completion time on the exponential function used with the average completion time as follows:

1. In the second Trapezium method. $3{,}30 \times 10^{-6}$
2. On the Romberg method of seconds.4,534

The completion time on functions with values and changes, lower and upper bounds, and can be seen in Table 6.$ax \cos bx \ \sin cx\ a, b\ c\ (a) = 2(b) = 5$

Table 6. Calculating time on the function $ax \cos bx \ \sin cx$

a	b	c	Iteration	Program execution time (second)	
				Trapecium Method	Romberg Method
3	6	7	10	$3{,}10 \times 10^{-6}$	0,007
4	6	7	13	$2{,}15 \times 10^{-6}$	0,062
2	5	7	15	$1{,}19 \times 10^{-6}$	0,263
2	8	7	17	$2{,}86 \times 10^{-6}$	0,967
6	10	8	20	$3{,}10 \times 10^{-6}$	8,205
6	12	8	21	$2{,}15 \times 10^{-6}$	15,227

The average completion time shown by Table 7 on the Trigonometric function is as follows:

1. In the second Trapezoidal method $3{,}24 \times 10^{-6}$
2. On the Romberg method of seconds.4,121

A comparison of the average completion time for each type of function can be seen in Table 7.

Table 7. Average turnaround time

Function types	**Trapecium Method**	**Romberg Method**
Polynomial	0,00000334	4,544
Exponential	0,00000302	4,535
Trigonometric	0,000002425	4,122
Average	**0,00000293**	**4,400**

Table 7 shows that the mean completion time on polynomial, exponential and trigonometric functions is always superior to the Trapesio method by a very significant comparison. The Trapesium method takes a very short time to solve the given problem, even though the function is a complex function. Unlike the Trapesium method, the Romberg method takes a long time to solve the given problem. The mean completion time on the Trapesium method and the Romberg method on all functions in a row is and seconds. $2{,}93 \times 10^{-6}$4,400

Conclusion

Based on the results of the research and discussion that has been carried out, using the PHP program, the average error in the Trapesium method is 9.497% with a minimum error of 0.178% on exponential function integrals, while in the Romberg method, the average error is obtained 0.979% with a minimum error of 0% on the integral of the polynomial function. This shows that the Romberg method is more accurate than the Trapezium method in approaching the exact value of the integral. However, the speed of completion time on the Trapesium method is much faster compared to the Romberg method. In the Trapezium method, the average completion time is seconds, and the average completion time in the Romberg method is seconds. The fastest turnaround time on the Trapesio method is $2{,}93 \times 10^{-6}4{,}4001{,}19 \times 10^{-6}$ seconds, and the fastest completion time on the Romberg method is 0,007 second.

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