GeoScienceEd *5(3) (2024)*

Jurnal Pendidikan, Sains, Geologi, dan Geofisika

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Comparison of Steffensen and Secant Methods in Determining Non-Linear Function Roots Using Hypertext Preprocessor (PHP) Programmer

Yasmin Mumtazi[1*], Amrullah[2], Nurul Hikmah[3], Sudi Prayitno[4]
[1]Mathematics Education Study Program, FKIP, University of Mataram, Mataram, Indonesia.

DOI: https://doi.org/10.29303/geoscienceed.v5i3.393

Article Info
Received: 01 August 2024
Revised: 23 August 2024
Accepted: 26 August 2024

Correspondence:

Phone: +6287865813936

Abstract: Mathematical modeling and simulation are more commonly found in everyday life ranging from simple to complex problems, in the context of modeling involving non-linear functions requires a theoretical understanding of non-linear equations. This study aims to compare the efficiency of Steffensen's method and Secant's method in determining the root of non-linear functions using PHP. The functions to be tested are polynomial functions, exponential functions, trigonometric functions, and mixed functions. The program test was carried out five times by changing the values of the coefficients of the function. Based on the results of the research and discussion, it was obtained that using the Secant method on polynomial functions, exponential functions and mixed functions, the number of iterations obtained was shorter with small errors and less time compared to the Steffensen method. Based on the results of the research and discussion, it was obtained that the Secant method is more efficiently used to find approximate values of polynomial functions, exponential functions, trigonometric functions, and mixed functions with a range of 0.1% - 0.5%. The average error in the Secant method is smaller than that of the Steffensen method in polynomial functions and exponential functions, while in trigonometry functions, the error of the Steffensen method is smaller than that of the Secant method. The execution time of the Secant method is less than that of the Steffensen method, so it can be concluded that the Secant method is more efficient than the Steffensen method in polynomial functions and exponential functions. Meanwhile, in trigonometric functions, the Steffensen method is more efficient compared to the Secant method.

Keywords: comparison; non-linear function; Steffensen method; Secant method; PHP.

Citation: Mumtazi, Y., Amrullah., Hikmah, N., & Prayitno, S. (2024). Comparison of Steffensen and Secant Methods in Determining Non-Linear Function Roots Using Hypertext Preprocessor (PHP) Programmer. *Jurnal Pendidikan, Sains, Geologi, dan Geofisika (GeoScienceEd Journal)*. 5(3), 418-424. Doi: https://doi.org/10.29303/geoscienceed.v5i3.393

Introduction

Modeling contexts involving non-linear functions require a theoretical understanding of non-linear equations. A non-linear function is a function whose curve is not in the form of a straight line. There are several forms of non-linear functions, including polynomial functions of at least two degrees, exponential functions, and trigonometric functions (Fanggidae, 2019).

In some problems, non-linear functions are often faced with the problem of finding the root of the function. In general, determining the roots of non-linear functions is more difficult than linear functions. According to Efendi & Subhan (2023), the solution of the root of non-linear functions is difficult to solve in an

Email: yasmiiinmumtazi02@gmail.com

analytical way because it has a unique shape and requires a complicated and time-consuming process. Therefore, other ways or methods are needed to solve these problems. In this case, the settlement can be done using numerical methods.

According to Ermawati, Rahayu, & Zuhairoh (2017) the numerical method is a method that uses repetitive computational calculations to solve numerical solutions and the results obtained are near the root of the non-linear function. In numerical methods, there are several algorithms to solve the root problem of a function, including Newton Raphson's method, Secant's method, and Steffensen's method. The Steffensen method and the Secant method are iterative methods developed from the Newton-Raphson method to solve non-linear functions (Putra, 2013).

Previous research has created several applications for solving the roots of functions numerically, such as Python, Pascal, C++, and PHP. However, the use of pascal, python, and java always applies offline so that the application cannot be accessed online, even though there is a programming language that can be used online, namely PHP. Therefore, this research will use the PHP programming language. PHP is designed to make it easier for beginner programmers to do their jobs because it is easy to understand and classified as a high-level programming language. In this case, PHP has an easy-to-understand syntax and supports more abstract programming features than low-level programming languages.

The purpose of this study is to obtain a program of the Steffensen method and the Secant method on the solution of non-linear function roots using PHP and obtain a comparison of the Steffensen method and the Secant method in terms of speed, iteration, and error in the numerical solution of polynomial functions, exponential functions, and trigonometry functions.

Method

This study uses a type of application research, which applies two algorithms from the Steffensen method and the Secant method applied in the PHP programming language to find the roots of non-linear functions reviewed from program execution time, number of iterations, and errors. The functions to be tested are polynomial functions, exponential functions, and trigonometric functions.

Steffensen Method Algorithm

The Steffensen method is defined as one of the methods derived and modified from Newton's method by estimating the derivative of the quotient of non-derivative terms (Sharma, 2005). The algorithm of the Steffensen method uses the PHP programming language as follows.

1. Specified Specifies the function for which the root will be sought.$f(x)$
2. Specifies maximum iteration and error tolerance.$(n)(\varepsilon)$
3. The value is determined as the initial guess.x_0
4. Given.$n = 0$
5. Calculated and. $f(x_n)f(x_n + f(x_n))$
6. Calculated using Steffensen's iterative:x_{n+1}
$$x_{n+1} = x_n - \frac{f(x_n)^2}{f(x_n + f(x_n)) - f(x_n)}$$
7. If the value is then written. If not, then proceed to step 4 by replacing it with. $|x_{n+1} - x_n| \leq \varepsilon x_{hampiran} = x_{n+1} x_{n+1} x_n$

Here is the Steffensen method$flow\ chart$

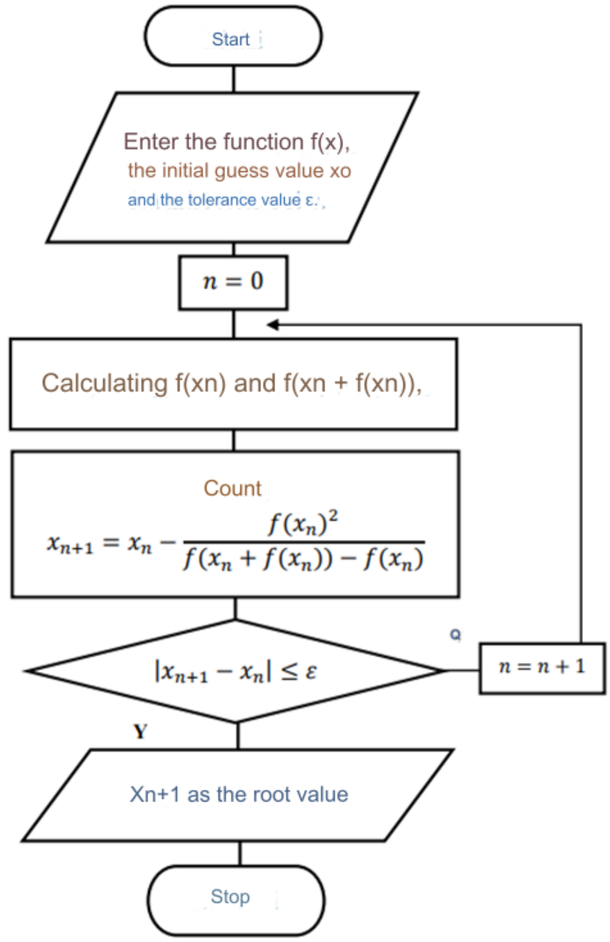

Figure 1. Flowchart of the Steffensen method

Secant Method Algorithm

The Secant method is a method that overcomes the weaknesses of Newton Raphson's method (Batarius & Sinlae, 2019) by utilizing two initial guessing values. The algorithm of the Secant method uses PHP as follows.

1. Defined functions to find the roots.$f(x)$
2. Maximum iteration and error tolerance are determined.(n) (ε)
3. Determined value and as two initial guess values.$x_0 x_1$
4. Calculated and. $f(x_0) f(x_1)$
5. Calculated by shapex_{n+1}

$$x_{n+1} = \frac{\left(x_{n-1}\, f(x_n) - x_n\, f(x_{n-1})\right)}{f(x_n) - f(x_{n-1})}, n = 1,2,3, \dots$$

6. If the value is then written. If not, then proceed to step 4 by replacing it with. $|x_{n+1} - x_n| \leq \varepsilon x_{hampiran} = x_{n+1} x_{n+1} x_n$

The following is the flow chart of the Secant method

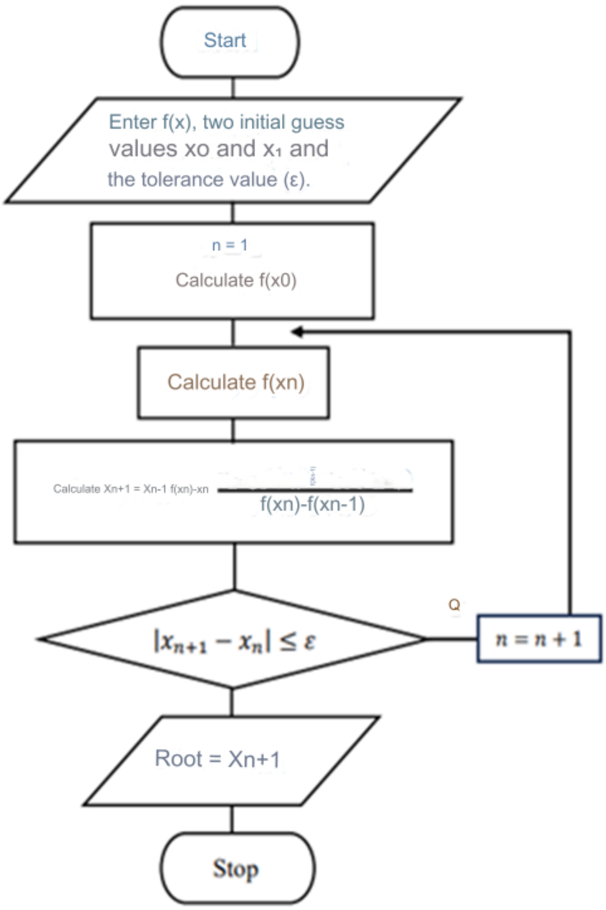

Figure 2. *Flowchart* method Secant

Results and Discussion

Comparison of the error of solving the roots of functions in the Steffensen method and the Secant method

Here are some of the functions that will be used in this research, including polynomial functions, exponential functions, trigonometric functions and mixed functions. The application program used is the PHP programming language of the Steffensen method and the Secant method by conducting validity tests based on several functions whose exact values are known as shown in Table 1.

Table 1 shows that for the polynomial function, the Secant method obtains an average error while the Steffensen method obtains an average error of. In the trigonometry function, the Secant method obtains an average error of . The results obtained show that the Secant method is more efficient in polynomial and exponential functions compared to the Steffensen method. However, in trigonometric function, the Steffensen method is more efficient than the Secant method. $5 \times 10^{-5} 1 \times 10^{-4}. 1 \times 10^{-4} 4{,}5 \times 10^{-4} 6 \times 10^{-4} 4{,}7 \times 10^{-4}$

Furthermore, an efficiency comparison between the Steffensen method and the Secant method was carried out by increasing the coefficients of each function. The functions in question are polynomial functions, exponential functions, trigonometric functions, and mixed functions.

Data retrieval from polynomial functions

Root determination data collection on polynomial functions with the format. With. Presented in $f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_m x^0 a_i = -1$ sampai 10 Table 2.

Data retrieval from exponential functions

In exponential functions, root determination data is taken on functions with the format. With up to 5. The results of the exponential function program of the Steffensen method and the Secant method are presented in $f(x) = a.e^{bx+d} - tb = 1$ Table 3.

Data retrieval from trigonometric functions

The format for taking root determination data on trigonometric functions is. With up to 5, up to 5. Presented in $f(x) = a_0 sin(b_0 x) + a_1 cos(b_1 x) + a_2 b_0 = 1 a_i = 1$ Table 4.

Data retrieval from mixed functions

Data was collected to determine the root of the mixed function with the format. With , to 5. The program results of the mixed functions are presented in $f(x) = \left(a_0 x e^{b_0 x^{n_1}} - a_1 sin^{n_2}(b_1 x + a_2 cos(b_2 x) + a_4\,)\right)^4 b_i = 1 a_i = -1$Table 5.

Based on the results of the program test, a graph was obtained between the coefficient and the average time needed to obtain the root result of the function in polynomial functions, exponential functions, and trigonometric functions.

Comparison graph of the solution time of the root of the polynomial function

Comparison of the speed of completion of the root of the polynomial function, with up to 11. The speed comparison graph is obtained as follows as shown in graph 1 below$f(x) = x^n + x^2 + 3x + 8n = 3$

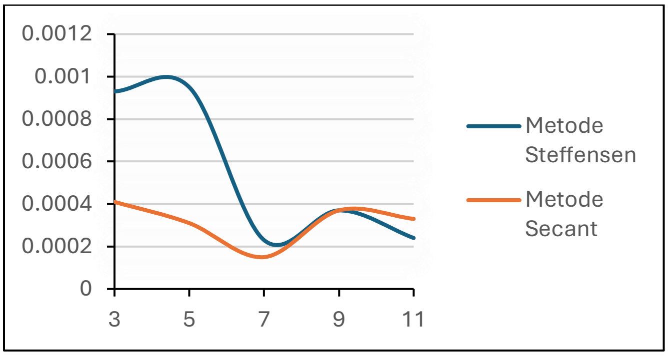

Figure 3. Time comparison of Steffensen's method and Secant's method

Exponential function root completion time comparison graph

Comparison of the completion speed of the root of the polynomial function f, with up to 5. The speed comparison graph is obtained as follows as shown in figure 2 below.$(x) = 3e^{bx+1} - 5b = 1$

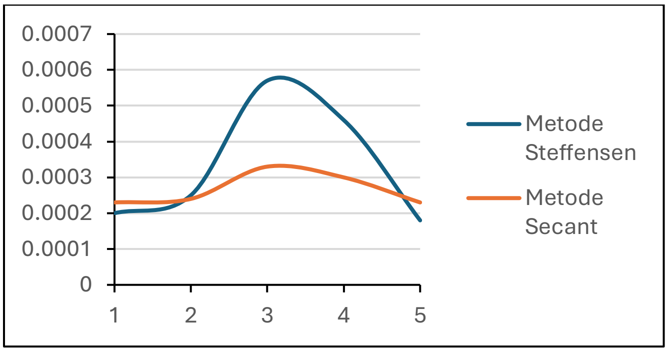

Figure 4. Time comparison of Steffensen's method and Secant's method

Comparison graph of the time of completion of the root of the trigonometric function

Comparison of the speed of completion of the root of the polynomial function, with up to 5. The speed comparison graph is obtained as follows as shown in figure 3 below.$f(x) = 2sin(b_0x) + cos\,(x) + 2b_0 = 1$.

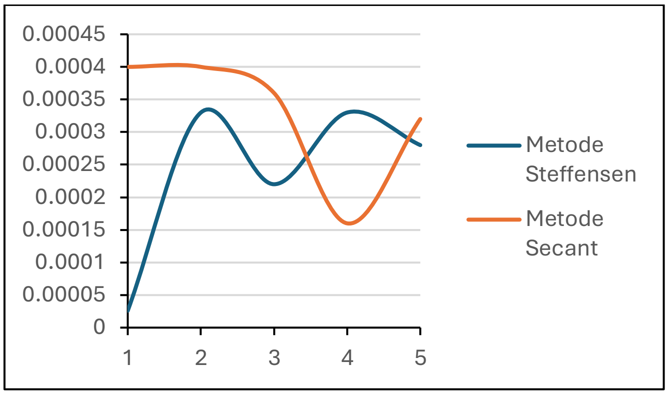

Figure 5. Time comparison of Steffensen's method and Secant's method

Table 1. Results of the Steffensen method and Secant method programs

It	Function	Exact	Steffensen Method			Secant Method		
			Iteration	Error	Numerical Results	Iteration	Error	Numerical Results
1	$f(x) = 4x^2 - 7x - 15$	$x_1 = -1{,}25$ $x_2 = 3$	9	0,0001	-1,249	6	0,0000	-1,249
2	$f(x) = x^3 - 9x^2 - 4x + 36$	$x_1 = -2$ $x_2 = 2$ $x_3 = 9$	55	0,0001	-2	7	0,0001	-1,999

3	$f(x) = 2e^{x} - 3$	0,405	5	0,0007	0,406	4	0,0002	0,405
4	$f(x) = e^{2x+1} - 5$	0,305	13	0,0002	0,305	4	0,0000	0,305
5	$f(x) = 4\,sin^2(x) - 1$	0,523	2	0,0000	0,523	2	0,0002	0,521
6	$f(x) = 2\,sin^2(x) - sin(x) - 1$	1,570	6	0,0006	1,570	10	0,0007	1,570
7	$f(x) = \left(xe^{x^2} - sin^2(x) + 3co\,s(x) + 5\right)^4$	-1,208	3	0,0008	-1,204	1	0,0009	-1,999

Table 2. Polynomial functions using the Steffensen method and the Secant method

It	Parameters							x_0	Steffensen Method			x_1	Secant Method		
	a_0	n	a_1	$n-1$	a_2	$n-2$	a_m		Iteration	Numerical Results	Time (milliseconds)		Iteration	Numerical Results	Time (milliseconds)
1	1	3	-1	2	3	1	8	0	11	-1,32024	0,00206	1	8	-1,31978	0,00065
2	2	3	0	2	3	1	8	0	18	-1,27735	0,00080	1	6	-1,27732	0,00021
3	2	3	5	2	4	1	9	0	47	-2,43712	0,00147	1	8	-2,43703	0,00055
4	3	3	5	2	5	1	9	1	85	-1,71551	0,00519	2	7	-1,71490	0,00035
5	3	3	5	2	5	1	10	1	96	-1,78200	0,00258	2	9	-1,78144	0,00046

Table 3. Exponential functions using Steffensen's method and Secant's method

It	Parameters				x_0	Steffensen Method			x_1	Secant Method		
	a	b	d	t		Iteration	Numerical Results	Time (milliseconds)		Iteration	Numerical Results	Time (milliseconds)
1	3	1	1	-5	0	8	-0,48916	0,00191	1	5	-0,48917	0,00074
2	3	2	1	-5	0	27	-0,24448	0,00116	1	5	-0,24458	0,00030
3	2	2	1	-5	0	3	-0,04176	0,00023	1	5	-0,04185	0,00030
4	1	3	2	-6	-0,1	5	-0,06908	0,00078	-2	5	-0,06941	0,00034
5	2	4	2	-6	-0,2	5	-0,22499	0,00035	-2	5	-0,22534	0,00020

Table 4. Trigonometric functions using the Steffensen method and the Secant method

It	Parameters					x_0	Steffensen Method			x_1	Secant Method		
	a_0	b_0	a_1	b_1	a_2		Iteration	Numerical Results	Time (milliseconds)		Iteration	Numerical Results	Time (milliseconds)
1	2	1	-1	1	1	-1	2	-2,21427	0,00139	-2	3	-2,21430	0,00022
2	3	2	1	1	1	-1	3	-1,36406	0,00034	-2	3	-1,36408	0,00017
3	3	3	1	1	2	-1	2	-1,08846	0,00021	-2	4	-1,08914	0,00035
4	4	4	2	1	2	-1	3	-1,19361	0,00018	-2	4	-1,19429	0,00038
5	5	5	-2	2	3	-0,1	4	-0,06793	0,00044	-2	7	-0,06839	0,00037

Table 5. The mixture function uses the Steffensen method and the Secant method

It	Parameters										x_0	Steffensen Method			x_1	Secant Method		
	a_0	b_0	n_1	a_1	n_2	b_1	a_3	b_2	a_4	n_3		Iteration	Numerical Results	Time (milliseconds)		Iteration	Numerical Results	Time (milliseconds)
1	1	1	2	-1	2	1	3	1	5	4	-1,2	3	-1,20426	0,00031	-2	1	-1,19999	0,00027
2	2	1	2	1	2	1	3	1	5	4	-1,1	3	-1,09396	0,00046	-2	1	-1,09999	0,00011
3	3	1	2	1	2	1	4	1	5	4	-1	2	-0,99216	0,00027	-2	1	-0,99999	0,00013
4	4	1	2	2	2	1	4	1	5	4	-0,9	0	-0,90000	0,00012	-2	1	-0,89999	0,00007
5	5	1	2	3	2	1	5	1	5	4	-0,9	2	-0,89656	0,00050	-2	1	-0,89999	0,00020

Discussion

Based on the above results, it is obtained that the Secant method has a smaller number of iterations, errors, and execution times compared to the Steffensen method. The order used in the function greatly affects the number of iterations in each method. This is in line with the opinion by Herfina, Amrullah, & Junaidi (2019) who said that the larger the order of functions used, the smaller the error obtained. And the opinion by Firdaus, Amrullah, Wulandari, & Hikmah (2023) is that the higher the iteration used, the longer the time needed to solve the problem. Therefore, the results obtained are more accurate.

The results in Table 2 for polynomial functions using the Steffesen method and the Secant method are shorter, when compared to the iteration for the Secant method than the Steffensen method. The average execution time required by the Secant method is faster than that of the Steffensen method. Figure 1 shows that the Secant method tends to require less time compared to the Secant method. The Secant method showed a lower execution time compared to the Steffensen method in the 3rd-7th order and showed better execution time stability with a smaller comparison after the 5th order. The Steffensen method shows a decrease in execution time in the 5th order, but overall the Steffensen method has a higher execution time. Thus, it can be concluded that the Secant method is more efficient than Steffensen's method on polynomial functions.

Table 3 for exponential functions shows that the number of iterations and execution time of the Secant method is less than that of the Steffensen method. With different coefficient values, the average time of the Secant method is 42% more efficient than the Steffensen method. The results obtained with only the value of b that increase can be seen in Figure 2 which shows that the time of the Secant method is lower compared to the Steffensen method. The Secant method shows better stability with a smaller ratio after. So it can be said that in the exponential function of the Secant method is 80% more efficient when viewed from its average time compared to the Steffensen method. $b_0 = 3$.

The test results for the trigonometric function in Table 4 show that the iteration of the Steffensen method is shorter than that of the Secant method. Meanwhile, the execution time of the Secant method is also less than that of the Steffensen method. In terms of the number of iterations, the Steffensen method is 67% more efficient than the Secant method. Judging from the execution time, the Secant method is 58% more efficient than the Steffensen method. Figure 3 shows that the execution time of the Secant method tends to be smaller at the value of Whereas the execution time of the Steffsensen method increases at , but not more than the execution time of the Secant method. Thus, it can be concluded that the Steffensen method is 86% more efficient than the Secant method for trigonometric functions. $b_0 =$ 3,4 hingga 5. $b_0 = 2$ dan $b_0 = 4$.

The results of the trial on mixed functions with different coefficients and initial values are shown in Table 5 it is obtained that the Secant method has fewer iterations and execution time compared to the Steffensen method. So, if you look at the number of iterations of the Secant method, it is 33% more efficient. When compared to the experiments that have been carried out by Romedian & Bustami (2014) where the method used is more thorough than the method in this study. However, the iteration used is longer than the method in this study. Therefore, it can be concluded that the Secant method is more efficient at polynomial, exponential and some mixed functions compared to the Steffensen method, whereas the Steffensen method is more efficient at trigonometric functions.

Based on the above results, it shows that the coefficients and initial values greatly affect the number of iterations and errors of each function. If the initial value is close enough to the actual value, then the number of iterations obtained is also small. On the other hand, if the initial value is far enough away from the actual value, then the root value is not converging. Coefficients and order also affect the number of iterations in this study. If the coefficients and orders are higher, then the number of iterations is less. A method can be said to be efficient if it gets the desired result with a smaller number of iterations and execution time. Judging from this problem, the Secant method is more efficient than the Steffensen method because obtaining the sought root value requires shorter iteration steps, with smaller errors, and faster time.

Conclusion

Based on the results of the Steffensen method and the Secant method using PHP in root determination, the following conclusions were obtained

1. The roots of the polynomial function obtained have an average error for the Secant method, while the Steffensen method of the obtained root has an average error of Further, the accuracy of the result was obtained that the Secant method is more accurate than the Steffesen method which is 99.99%. 5×10^{-5} 1×10^{-4}.
2. The root of the exponential function in the Secant method obtains an average error, i.e., while the Steffensen method has an average error. The accuracy of the results obtained that the Secant method is more accurate than the Steffensen method which is 99.955%. 1×10^{-4} $4{,}5 \times 10^{-4}$
3. The root of the trigonometric function obtained the mean error for the Steffensen method, while the Secant method obtained the average error The accuracy of the results obtained for the Secant method was more accurate than the Steffensen method which was 99.94%. $4{,}7 \times 10^{-4}$ 6×10^{-4}.
4. The execution time of the Secant method is faster compared to the Steffensen method. For

polynomial functions the Secant method has an accuracy of 30%, for exponential functions the Secant method has an accuracy of 28% and for trigonometric functions the Secant method has an accuracy of 97% compared to the Steffensen method.

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